MATHEMATICS (SYLLABUS D)   **4024/02**

Paper 2   **October/November 2009**

2 hours 30 minutes

Additional Materials: Answer Booklet/Paper   Graph paper (2 sheets)
                        Electronic calculator   Mathematical tables (optional)
                        Geometrical instruments

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Section A
Answer **all** questions.

Section B
Answer any **four** questions.

Show all your working on the same page as the rest of the answer.
Omission of essential working will result in loss of marks.
You are expected to use an electronic calculator to evaluate explicit numerical expressions. You may use mathematical tables as well if necessary.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 100.

This document consists of **10** printed pages and **2** blank pages.

[Turn over

Section A [52 marks]

Answer **all** questions in this section.

1   Solve the equations

   (a) $2^y = 8$,   [1]

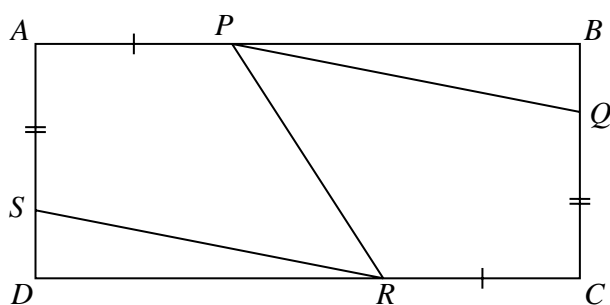   **(b)** $3p + 4 = 8 - 2(p - 3)$,   [2]

   (c) $\dfrac{18}{q} - \dfrac{16}{q + 2} = 1$,   [3]

   (d) $5x^2 + x - 7 = 0$, giving each solution correct to 2 decimal places.   [4]

2

ABCD is a rectangle.
Points *P*, *Q*, *R* and *S* lie on *AB*, *BC*, *CD* and *DA* respectively such that *AP = CR* and *QC = SA*.

   (a)   Giving reasons, show that

      (i)   *PB = RD*,   [1]

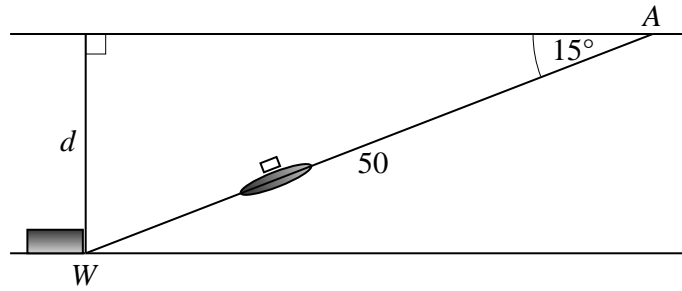   **(ii)**   triangle *PBQ* is congruent to triangle *RDS*,   [3]

      (iii) $R\hat{P}Q = P\hat{R}S$.   [3]

   (b)   State the special name of the quadrilateral *PQRS*.   [1]

3   A small submarine dived in a straight line from a point *A* on the surface to examine an object at the point *W* on the seabed.

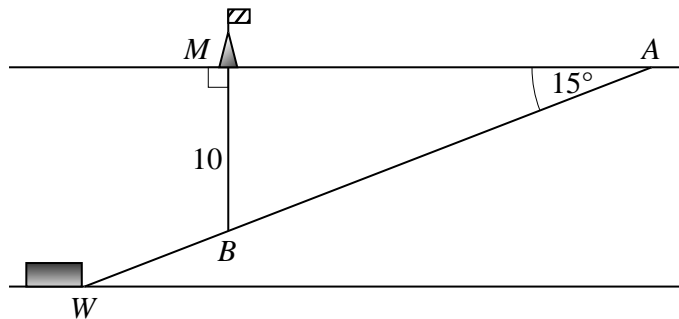**(a)**

It dived at an angle of 15° to the horizontal and reached *W* after travelling 50 m.

Calculate the depth, *d* metres, of the seabed at *W*.   [2]

(b)

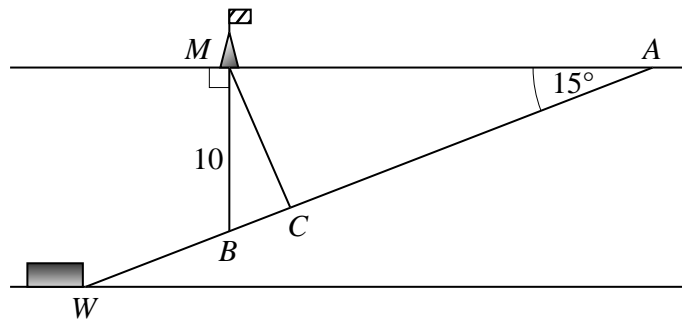A marker is at the point *M* on the surface.
When at *B*, the submarine was 10 m vertically below *M*.

Calculate *AB*.   [3]

(c)

When at *C*, the submarine was at its nearest point to *M*.

(i) Find $B\hat{M}C$.   [1]

(ii) Calculate *CM*.   [2]

4 **(a)** $\mathscr{E} = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$

$L = \{x : x$ is an odd number$\}$

$M = \{x : x$ is a multiple of 3$\}$

(i) Write down

(a) $L \cap M$, [1]

(b) $L' \cap M$. [1]

(ii) A number n is chosen at random from $\mathscr{E}$.
Find the probability that $n \in L \cup M$. [1]

(b) In a survey, a number of people were asked "Do you own a car?" and "Do you own a bicycle?".
The Venn diagram shows the set C of car owners and the set B of bicycle owners.
The letters p, q and x are the numbers of people in each subset.
11 people owned neither a car nor a bicycle.
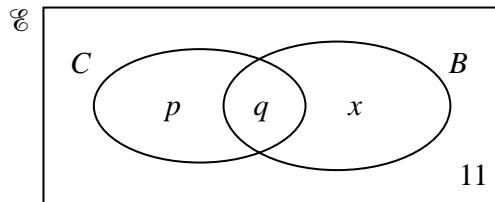
A total of 66 people owned a car.
4 times as many people owned a car **only** as owned a bicycle **only**.

(i) Write down expressions, in terms of x, for

(a) p, [1]

(b) q. [1]

(ii) A total of 27 people owned a bicycle.

Calculate

(a) x, [2]

(b) the **total** number of people who were in the survey. [1]

5 **(a)** Evaluate

    (i) $\quad 3\begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix}$, [2]

    (ii) $\quad (1 \ \ 3 \ \ 4)\begin{pmatrix} 0 & 4 \\ 3 & 1 \\ 5 & 0 \end{pmatrix}$. [2]

(b) $\quad \mathbf{A} = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}$

    (i) Find $\mathbf{A}^{-1}$. [2]

    (ii) The transformation represented by the matrix **A** maps (h,k) onto $(10,2)$.

        Find the value of h and the value of k. [2]

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6 **(a)** 100 g of spaghetti contains 3.6 g of fibre.

    Express

                mass of fibre : mass of spaghetti

    as the ratio of two integers in its simplest form. [1]

(b) A tin contains 210 g of beans.

    (i) 100 g of beans contains 4.5 g of protein.

        Calculate the mass of protein in the tin. [1]

    (ii) 100 g of beans contains 0.3 g of fat.

        (a) What percentage of the beans is fat? [1]

        (b) The recommended daily amount of fat is 70 g.

            Calculate what percentage of the recommended daily amount is in the tin. [3]

    (iii) The mass of salt in 100 g of beans is 1.0 g, correct to 1 decimal place.

        Calculate an upper bound for the mass of salt contained in the tin. [2]

(c) A tin of soup contains 166 calories.
This is 8.3% of the recommended daily number of calories.

    Calculate the recommended daily number of calories. [2]

Section B [48 marks]
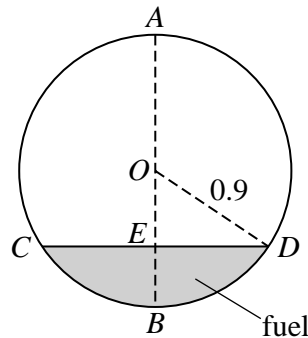
Answer **four** questions in this section.

Each question in this section carries 12 marks.

7 **(a)** A fuel tank is a cylinder of diameter 1.8 m.

    (i) The tank holds 25 000 litres when full.

       Given that $1m^3 = 1000$ litres, calculate the length of the cylinder.
       Give your answer in metres. [4]

    (ii)

       The diagram shows the cross-section of the cylinder, centre O, containing some fuel.
       CD is horizontal and is the level of the fuel in the cylinder.
       AB is a vertical diameter and intersects CD at E.
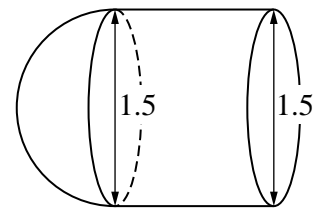
       Given that E is the midpoint of OB,

       (a) show that $E\hat{O}D = 60°$, [1]

       (b) calculate the area of the segment BCD, [3]

       (c) calculate the number of litres of fuel in the cylinder. [2]

(b) **[Volume of a sphere $= \frac{4}{3}\pi r^3$]**

    A different fuel tank consists of a cylinder of diameter 1.5 m
    and a hemisphere of diameter 1.5 m at one end.

    The volume of the cylinder is 10 times the volume of the hemisphere.

    Calculate the length of the cylinder. [2]

**8** **Answer the whole of this question on a sheet of graph paper.**

(a) The variables x and y are connected by the equation

$$y = 4x^3 - 18x^2 + 20x.$$

The table below shows some values of x and the corresponding values of y.

| x	0	0.5	1	1.5	2	2.5	3	3.5
y	0	6	6	3	0	0	6	p

   (i) Calculate the value of p. [1]

   (ii) Using a scale of 2 cm to represent 1 unit, draw a horizontal x-axis for $0 \leqslant x \leqslant 4$.
   Using a scale of 1 cm to represent 2 units, draw a vertical y-axis for $-4 \leqslant y \leqslant 24$.
   On your axes, plot the points given in the table and join them with a smooth curve. [3]

   (iii) Using your graph, find the values of x when $y = 4$. [2]

(b) A rectangular card is 5 cm long and 4 cm wide.
   As shown in the diagram, a square of side x centimetres is cut off from each corner.
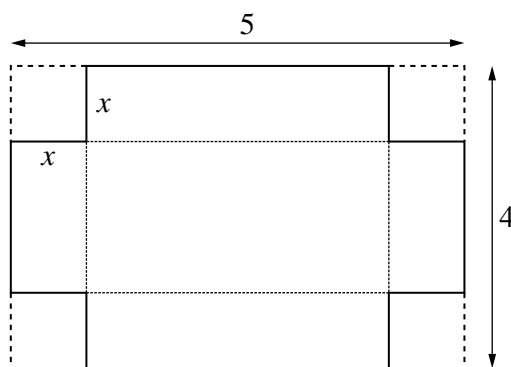
The card is then folded to make an open box of height x centimetres.

   (i) Write down expressions, in terms of x, for the length and width of the box. [1]

   (ii) Show that the volume, V cubic centimetres, of the box is given by the equation

$$V = 4x^3 - 18x^2 + 20x.$$ [2]

   (iii) Which value of x found in **(a)(iii)** **cannot** be the height of a box with a volume of $4\,\text{cm}^3$? [1]

   (iv) Using the graph drawn in part **(a)(ii)**, find

   (a) the greatest possible volume of a box made from this card, [1]

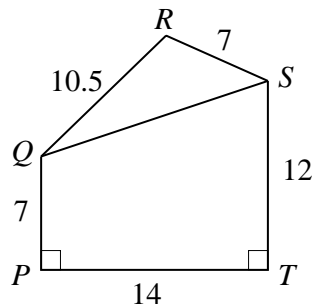   (b) the height of the box with the greatest volume. [1]

9 **(a)** The diagram shows the side *ABCDE* of a building.
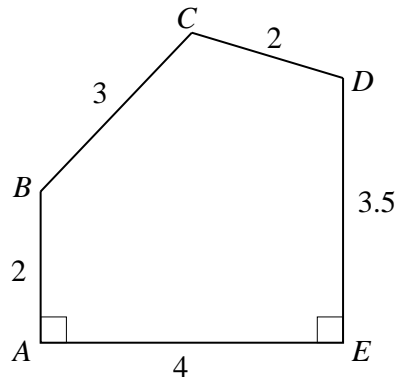
$AB = 2$ m, $BC = 3$ m, $CD = 2$ m, $DE = 3.5$ m and $EA = 4$ m.
AB and *DE* are vertical.
AE is horizontal.

   (i) Using a scale of 2 cm to 1 m, construct an accurate scale drawing of *ABCDE*. [3]

   (ii) Measure and write down $A\hat{B}C$. [1]

(b) The diagram shows the pentagon *PQRST*.

$PQ = 7$ cm, $QR = 10.5$ cm, $RS = 7$ cm, $ST = 12$ cm and $TP = 14$ cm.
$Q\hat{P}T = P\hat{T}S = 90°$.

   (i) Explain why the shapes *ABCDE* and *PQRST* are not geometrically similar. [1]

   (ii) Show that $QS^2 = 221$. [2]

   (iii) Calculate $Q\hat{R}S$. [3]

   (iv) Calculate $R\hat{Q}S$. [2]

10 Answer the whole of this question on a sheet of graph paper.

80 electric light bulbs of brand A were tested to find how long each bulb lasted.
The results are summarised in the table below.

Time (*t* hours)	$t \leqslant 50$	$50 < t \leqslant 100$	$100 < t \leqslant 150$	$150 < t \leqslant 200$	$200 < t \leqslant 250$	$250 < t \leqslant 300$	$300 < t \leqslant 350$	$350 < t \leqslant 400$
Number of bulbs	1	2	6	34	26	8	2	1

(a) Copy and complete the following cumulative frequency table.

Time (*t* hours)	$t \leqslant 50$	$t \leqslant 100$	$t \leqslant 150$	$t \leqslant 200$	$t \leqslant 250$	$t \leqslant 300$	$t \leqslant 350$	$t \leqslant 400$
Number of bulbs	1	3						80

[1]

(b) Using a horizontal scale of 2 cm to represent 50 hours and a vertical scale of 2 cm to represent 10 bulbs, draw a smooth cumulative frequency curve for these brand A bulbs. [3]

(c) Use your graph to estimate

    (i) the median, [1]

    (ii) the 10th percentile. [1]

(d) 80 brand B bulbs were also tested and a report on the test gave the following information.

    3 bulbs lasted 50 hours or less.
    No bulbs lasted more than 350 hours.
    The median time was 250 hours.
    The upper quartile was 275 hours.
    The interquartile range was 75 hours.

    On the same axes, draw a smooth cumulative frequency curve for the brand B bulbs. [3]
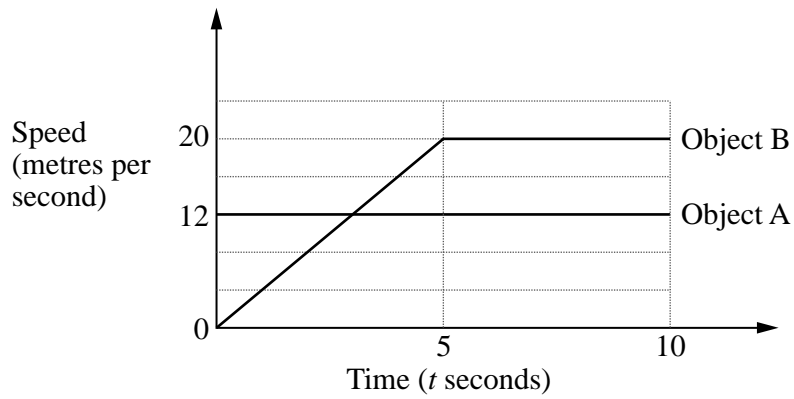
(e) Use your graphs to estimate the number of bulbs that lasted 260 hours or less

    (i) for brand A, [1]

    (ii) for brand B. [1]

(f) Which brand of bulb is more likely to last longer than 250 hours?
Justify your answer. [1]

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The diagram shows the speed-time graphs of two objects, A and B, for the first 10 seconds of their motion.

Object A travelled at a constant speed of 12 m/s throughout the 10 seconds.

Object B started from rest, and accelerated at a constant rate, attaining a speed of 20 m/s after 5 seconds. It then travelled at a constant speed of 20 m/s.

(a) Calculate

    (i) the distance travelled by object B during the first 5 seconds of its motion, [1]

    (ii) the average speed of object B for the first 10 seconds of its motion, [2]

    (iii) the value of t when both objects were travelling at the same speed, [2]

    (iv) the value of t when both objects had travelled the same distance. [2]

(b) The diagram below shows the distance-time graph for object B.
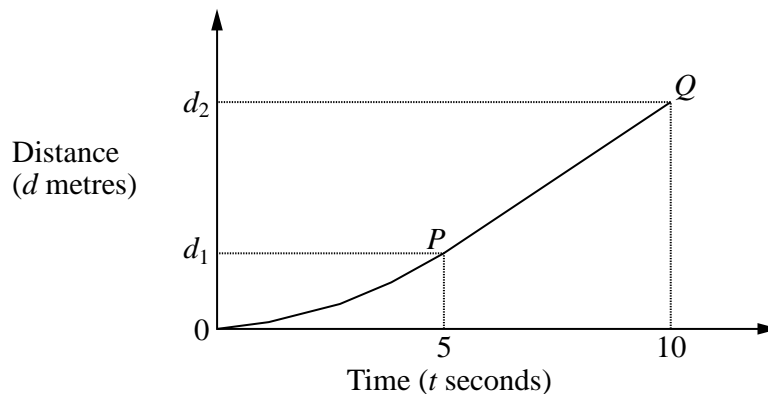
In the diagram, OP is a curve and PQ is a straight line.

    (i) State the values of d_1 and d_2. [1]

    (ii) What does the gradient of the straight line PQ represent? [1]

    (iii) Write down the gradient of the tangent to the curve at $t = 2.5$. [1]

(c) After 10 seconds, both objects slowed down at the same constant rate.
Object *A* came to rest after a further 9 seconds.
After how many seconds **from the start of its motion** did object *B* come to rest? [2]

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